

Comparison of the Centroid Method and Four Standard Formulas for Estimating Log Volumes

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Abstract: Centroid Sampling was tested on 21 logs of Ash (*Fraxinus angustifolia* subsp. *oxycarpa*), 38 logs of Spruce (*Picea orientalis* (L.) Link.), and 33 logs of Beech (*Fagus orientalis* Lipsky.) all of which were measured in detail. The volume of each log was estimated using Huber's, Smalian's, Newton, Riecke's, Hosfeld's formulas and Centroid Sampling. These estimates were compared with "true" volume of each log which was determined by aggregating the volumes of measured short sections (1 m.) using Smalian's formula.

The mean error of the Centroid estimate of the log volumes was not significant for *Fraxinus angustifolia* subsp. *oxycarpa*, *Picea orientalis* (L.) Link., and *Fagus orientalis* Lipsky. and was less than those derived from Huber's, Smalian's, Newton-Riecke's, and Hosfeld's formulas. When three species were combined, the Centroid estimate was clearly more accurate, and its mean error was not significant at 0.05 probability level.

Tomruk Hacimlerinin Hesaplanmasında Centroid Metod ve Dört Standart Formülün Kıyaslanması

Özet: Ayrıntılı bir şekilde ölçülen 21 Dişbudak (*Fraxinus angustifolia* subsp. *oxycarpa*), 38 Ladin (*Picea orientalis* (L.) Link.) ve 33 Kayın (*Fagus orientalis* Lipsky.) tomruğuna bağlı olarak Centroid Sampling yöntemi test edilmiştir. Her bir tomruğa ilişkin hacimler; Huber, Smalian, Newton-Riecke, Hosfeld ve Centrod yöntemleriyle ayrı ayrı hesaplanarak tomrukların gerçek hacim değerleri ile kıyaslanmıştır. Tomrukların gerçek hacim değerleri ise Smalian yöntemi kullanılarak 1'er metrelik seksiyonların hacimleri toplamı olarak hesaplanmıştır.

Centroid yöntemine göre tomruk hacimlerinin ortalama hatası her üç ağaç türü için de sıfırdan farksız bulunmuştur ($p>0.05$) ve en düşük ortalama hata değeri Centroid yöntemiyle elde edilmiştir. Üç ağaç türü birleştirildiğinde; Centroid yöntemi daha doğru sonuç vermiştir ve ortalama hatası 0.05 önem düzeyi ile sıfırdan farksız bulunmuştur.

Introduction

Generally, for estimating log volumes some standard formulas, such as Huber's, Smalian's, Hosfeld's, Simoney's, and Newton-Riecke's formulas have been used (1, 2, 3). From these formulas, because of simplicity and practicality the Huber's formula is frequently used for log volume estimation.

A new approach (Importance Sampling) was developed by Gregoire et. al. (4) to predict log volumes. It uses an assumed taper function ("proxy function"). The volume calculated by the proxy function is adjusted by the ratio of actual cross-sectional area at a randomly selected point on the log to the cross-sectional area of that point as predicted by the proxy function. On the other hand, Centroid method (Centroid Sampling), a variant of Importance Sampling was developed by Wood et al. (5) for estimating log or tree volumes.

Using the Centroid Sampling, some researches (5, 6, 7, 8, 9, 10) obtained reliable results than some standard

formulas (e. g., Huber, Smalian, and Newton-Riecke) for various tree species. This study compared the bias of the Centroid Method and four standard formulas (Huber, Smalian, Hosfeld, and Newton-Riecke) for estimating the volume of 6-meter logs from the stump upwards. This particular investigation used the data obtained from the boles of *Fraxinus angustifolia* subsp. *oxycarpa*, *Picea orientalis* (L.) Link., and *Fagus orientalis* Lipsky.

Material and Methods

Twenty-one trees of Ash (*Fraxinus angustifolia* subsp. *oxycarpa*), thirty-eight trees of Spruce (*Picea orientalis* (L.) Link.), and thirty-three trees of Beech (*Fagus orientalis* Lipsky.) were felled. Their diameter at breast height varied from 12.2 to 85.8 cm., and total height ranged from 11.4 to 36.7 m. (Table 1). Two 6-meter logs were cut from each tree. Diameter outside bark ($d_{b\text{thob}}$) was measured with caliper at 1m. intervals along the length of each log.

Table 1. The Sample Tree Data Base Used to Compare the Methods of Estimating Log Volume

Species	No. of trees	Diameter over bark		Total height		Forest type	Location
		Mean(cm)	S.D.(cm)	Mean (m)	S.D.(m)		
Ash	21	31.8	8.8	19.4	4.9	Even-aged	Sinop
Beech	33	53.1	13.3	24.1	7.6	Even-aged	Trabzon
Spruce	38	36.8	9.2	20.6	5.4	Even-aged	Trabzon
All species	92	41.5	10.8	21.6	6.2		

Log volumes were calculated for 6 m. log length using the Huber, Smalian, Hosfeld, Newton-Riecke, and Centroid methods given in Equation (1), (2), (3), (4), and (5), respectively. They were then compared to the corresponding accumulated volumes of 1-meter sections calculated using Smalian's formula, the assumed true volume.

$$\text{Huber: } V = ML \tag{1}$$

$$\text{Smalian: } V = ((B+S)/2)L \tag{2}$$

$$\text{Hosfeld: } V = ((3G + S)/4)L \tag{3}$$

$$\text{Newton-Riecke: } V = ((B+4M + S)/6)L \tag{4}$$

$$\text{Centroid: } V = SL + (1/2)b_1L^2+(1/3)b_2L^3 \tag{5}$$

Where

$$b_1 = (B-S-b_2L^2)/L \tag{6}$$

$$b_2 = (B-C(L/e)-S(1-L/e))/(L^2-Le) \tag{7}$$

B = Cross-sectional area at large end of log (m²)

G = Cross-sectional area at 1/3 of log length from the large end of the log (m²)

M = Cross-sectional area at mid-length of log (m²)

S = Cross-sectional area at small end of log (m²)

L = long length (m.)

C = Cross-sectional area at mid volume of log (m²) measured at a distance q from the large end of log. Where:

$$q = L \frac{\sqrt{(d_0/d_n)^4 - \sqrt{2}}}{((d_0/d_n)^2 - 1)\sqrt{2}} \tag{8}$$

$$e = L-q$$

d₀, d_n = diameter (cm) at large and small end of log, respectively.

In the Centroid method, the log volume is estimated by three steps. At the first step, diameter at large (d₀) and small (d_n) end of the log, and the log length (L) are measured. At the second step, the Centroid distance (q) from

the large end of the log is calculated by Equation (8), and at this point the Centroid diameter (d_c) is measured. And, at the last step, the parameters (b₁ and b₂) of the Centroid Volume Equation (5) are estimated by Equation (6) and (7). The theory of this method can be obtained from the literature (5), and a computer program written using Basic Programming Language can be obtained from the literature (11).

Data were analyzed using LOGVOL Program given in Appendix 1. Calculations determined whether mean error or bias (Equation 9) was significantly different from zero by using Equation (10).

$$\text{Bias} = (1/n)(\Sigma(V_i^* - V_i)) \tag{9}$$

$$t \text{ - statistics} = \left(\frac{\text{Bias}}{\text{S.D. of (Bias)}} \right) \sqrt{n} \tag{10}$$

Where

V_i^{*} = Predicted log volume

V_i = True log volume

n = Number of logs

S.D. = Standard deviation.

Results and Discussion

For each of the three species tested, the Centroid method gave less biased estimates of 6-meter log volumes than the standard formulas (Table 2 and 3). For all species, the mean error of the lower bole (butt 6 m.) derived Huber's and Smalian's formula gave the least accurate results. The mean error of the Huber's and Smalian's estimate were significant at the 0.05 probability level (Table 2). For the second 6-meter log of all species, Centroid's and Hosfeld's method produced the least biased result, and the mean errors of the other standard estimates (Huber's, Smalian's, and Newton-Riecke's method) were significant at 0.05 probability level for all species (Table 3).

Table 2. Statistics for Comparing Estimates of Butt 6-meter Log Volume Derived Using Huber's, Smalian's, Hosfeld's, Newton-Riecke's, and Centroid Method

Species	No. of logs	Method	Mean vol. (m3)	Mean error (m3)	S. D. of errors (m3)	t-statistics	Probability (*)
Ash	21	True	.3838				
		Huber	.3612	-.0226	.028	-3.699	0.00142
		Smalian	.4360	.0522	.045	5.316	0.00003
		Newton	.3901	.0063	.022	1.312	0.20438
		Hosfeld	.3630	-.0208	.025	-3.813	0.00109
		Centroid	.3813	-.0025	.019	-0.603	0.55329
Spruce	38	True	.4691				
		Huber	.4581	-.0110	.013	-5.216	0.00001
		Smalian	.5072	.0381	.082	2.864	0.00685
		Newton	.4613	-.0078	.020	-2.404	0.02134
		Hosfeld	.4781	.0090	.029	1.913	0.06351
		Centroid	.4732	.0041	.015	1.645	0.10844
Beech	33	True	1.1542				
		Huber	1.1457	-.0085	.041	-1.191	0.24241
		Smalian	1.1763	.0221	.044	2.885	0.00695
		Newton	1.1481	-.0061	.021	-1.669	0.10487
		Hosfeld	1.1447	-.0095	.024	-2.274	0.02981
		Centroid	1.1501	-.0041	.018	-1.308	0.20019
All	92	True	.6953				
		Huber	.6824	-.0129	.029	-4.267	0.00005
		Smalian	.7308	.0355	.063	5.405	0.00000
		Newton	.6914	-.0039	.021	-1.781	0.07825
		Hosfeld	.6907	-.0046	.026	-1.697	0.09311
		Centroid	.6949	-.0004	.017	-0.226	0.82171

(*) Null hypothesis (Ho): Mean error = 0

Table 3. Statistics for Comparing Estimates of Second 6-meter Log Volume Derived Using Huber's, Smalian's, Hosfeld's, Newton-Riecke's, and Centroid Method

Species	No. of logs	Method	Mean vol. (m3)	Mean error (m3)	S. D. of errors (m3)	t-statistics	Probability (*)
Ash	21	True	.2511				
		Huber	.2572	-.0061	.011	2.541	0.01945
		Smalian	.2585	.0074	.014	2.422	0.02402
		Newton	.2556	.0045	.012	1.718	0.10124
		Hosfeld	.2457	-.0054	.013	-1.904	0.07140
		Centroid	.2525	-.0014	.009	-0.713	0.48408
Spruce	38	True	.3325				
		Huber	.3152	-.0083	.032	-1.599	0.11833
		Smalian	.3375	.0140	.039	2.213	0.03315
		Newton	.3214	-.0021	.011	-1.177	0.24671
		Hosfeld	.3161	.0074	.030	-1.521	0.13676
		Centroid	.3222	-.0013	.007	-1.145	0.25956
Beech	33	True	0.8172				
		Huber	.7955	-.0217	.029	-4.299	0.00015
		Smalian	.8365	.0193	.032	3.465	0.00153
		Newton	.8051	-.0121	.024	-2.896	0.00676
		Hosfeld	.8262	.0090	.024	2.154	0.03888
		Centroid	.8203	.0031	.017	1.048	0.30249
All	92	True	.4840				
		Huber	.4742	-.0098	.027	-3.481	0.00077
		Smalian	.4985	.0145	.032	4.346	0.00004
		Newton	.4799	-.0041	.017	-2.313	0.02298
		Hosfeld	.4830	-.0010	.025	-0.384	0.70187
		Centroid	.4849	-.0009	.012	-0.719	0.47398

(*) Null hypothesis (Ho): Mean error = 0

When all species were combined, the Centroid estimate was clearly more accurate than those of Huber, Smalian, Newton-Riecke, and Hosfeld, its mean error being nonsignificant ($p > 0.05$). But, the Centroid method is more costly to apply than Huber's or Smalian's because it requires three diameter measurements, one at each end and one at the Centroid, but it is no more costly than Newton-Riecke's, and Hosfeld's formula. The time

required to apply each method is identical but determining the Centroid position on a log requires a pocket sized calculator to compute Equation (8).

These results indicate that the Centroid method is a useful alternative to other standard formulas. Thus, this study reinforces the notion that the Centroid of a log defines a position of special significance for estimating its volume.

Appendix 1. LOGVOL Program

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10 INPUT "Enter the number of logs": N
20 INPUT "Enter the number of measured diameters on each log" : M
30 INPUT "Enter section length (m.)": SL
40 REM DB (cm), DG (cm) DC (cm), DM (cm), DS (cm) are diameters at large end,
50 REM L/3 from large end, Centroid, middle, and small end of log.
60 REM Q (M) is the mid-volume (Centroid) distance from the small end of
70 REM the log and L (M) is the log length.
80 DIM VTRUE (N), VHUB (N), VSMAL (N), VHOS (N), VNEW (N), VCENT (N)
90 DIM DB (N), DG (N), DM (N), DC (N), DS (N), L (N), Q (N), D (N, M)
100 REM Reading the data.
110 FOR I= 1 TO N
120 READ DB (I), DG (I), DC (I), DM (I), DS (I), L (I), Q (I)
130 DB (I) = DB (I)/100 : DG (I) = DG (I)/100: DC(I)=DC (I)/100: DM (I) = DM (I) /100
140 DS (I) = DS (I) /100:
150 NEXT I
160 FOR I=1 TO N
170 FOR J=1 TO M
180 READ D (I, J)
190 D (I, J) = D(I, J) / 100
200 NEXT J, I
210 REM Calculating the true volumes (VTRUE) using the Smalian formula.
220 FOR I= 1 TO N
230 A= (D(I, 1)^2+D(I, M)^2) / 2
240 FOR J= 2 TO M-1
250 A=A+D (I, J)^2
260 NEXT J
270 VTRUE (I) = A*.78513*SL
280 A=0
290 NEXT I
300 PI=3.141528:K=PI/4
310 PRINT " LOG VOLUMES (m3) "
320 PRINT TAB (1) "No of LogS" TAB (13)"VTRUE" : TAB (23) "VHUB": TAB (33) "VSMAL": TAB (43) "
VHOS": TAB (53)"VNEW": TAB (63)"VCENT"
330 REM Calculating the log volumes using the Huber (VHUB), Smalian (VSMAL),
340 REM Hosfeld (VHOS), Newton-Riecke (VNEW) formulas, and Centroid Sampling
350 REM (VCENT).
360 FOR I=1 TO N
370 VHUB (I) = K*DM (I)^2*L(I)
380 VSMAL (I) = (K* (DB (I)^2+DS (I)^2)/2)*L(I)
390 VHOS (I) = (K*(4*DG (I)^2 + DS (I)^2) / 4)*L (I)
400 VNEW (I) = (K* (DB (I)^2+4*DM(I)^2+DS(I)^2)/6)* L (I)
410 E=L (I) - Q (I)
420 B2 = (K*DB (I)^2-K*DC (I)^2* (L (I) /E)- K*DS (I)^2* (1-L (I) / E))
430 B2=B2 / (L (I)^2-L (I)*E)
440 B1 = (K*DB (I)^2-K*DS (I)^2-B2*L (I)^2) / L (I)
450 VCENT (I) = K*DS (I)^2*L (I) + (1/2)*B1*L(I)^2+(1/3)*B2*L (I)^3
460 PRINT TAB (4) I: TAB (10) VTRUE (I): TAB (20) VHUB (I): TAB (30) VSMAL (I): TAB (40) VHOS (I): TAB (50) VNEW (I): TAB (60) VCENT (I)
470 NEXT I
480 PRINT
490 REM Calculating the bias for each formula.

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500 E1 = 0: E2 = 0: F1 = 0: F2 = 0: G1 = 0: H1 = 0: H2 = 0: J1 = 0: J2 = 0
510 PRINT "          RESIDUALS OF THE LOG VOLUMES (m3) "
520 PRINT TAB (1) "No of Log"; TAB (13)" RHUB "; TAB (23)" RSMAL"; TAB (33) "RHOS"; TAB (43)"RNEW" ; TAB (53) "RCENT"
530 FOR I = 1 TO N
540 W1 = VHUB (I) - VTRUE (I) : E1 = E1 + W1: E2 = E2 + W1^2
550 W2 = VSMAL (I) - VTRUE (I) : F1 = F1 + W2: F2 = F2 + W2^2
560 W3 = VHOS (I) -VTRUE (I) : G1 = G1 + W3 : G2 = G2 + W3^2
570 W4 = VVEW (I) - VTRUE (I) : H1 = H1 + W4 : H2 = H2 +W4^2
580 W5 = VCENT (I) - VTRUE (I) : J1 = J1 +W5 : J2 = J2 + W5^2
590 PRINT USING"#####      #####      #####      #####      #####      #####n" : I, W1, W2, W3, W4, W5
600 NEXT I
610 PRINT
620 PRINT "          AVERAGE RESIDUALS OF THE METHODS"
630 PRINT TAB (13) "RHUB "; TAB (23) "RSMAL"; TAB (33) "RHOS"; TAB (43) "RNEW"; TAB (53)" RCEN T"
640 PRINT USING"#####      #####      #####      #####      #####      #####n" : E1/N, F1/N, G1/N, H1/N, J1/N
650 SHUB = SQR ( (E2 - E1^2/N) / (N-1))
660 SSMAL = SQR ( (F2 - F1^2/N) / (N-1))
670 SHOS = SQR ( (G2 - G1^2/N) / (N-1))
680 SNEW = SQR ( (H2 - H1^2/N) / (N-1))
690 SCENT = SQR ( (J2 - J1^2/N) / (N-1))
700 KK = SQR (N)
710 REM Calculating the Student's t-statistics for eacj formula (HO: bias = 0.
720 REM Ha: bias not equal 0).
730 THUB = ( (E1 / N)*KK) / SHUB
740 TSMAL = ( (F1 / N)*KK) / SSMAL
750 THOS = ( (G1 / N)*KK) / SHOS
760 TNEW = ( (H1/N)*KK) / SNEW
770 TCENT = ( (J1/N)*KK) / SCENT
780 PRINT
790 PRINT "          t-statistics of the methods          "
800 PRINT TAB (13) " t-HUB "; TAB (23) " t-SMAL" ; TAB (33) " t-HOS"; TAB (43) " t-NEW" ; TAB (53) " t-CENT"
810 PRINT USING" ##.#####      ##.#####      ##.#####      ##.#####      ##.#####      ##.#####n" : THUB, TSMAL, THOS, TNEW, TCENT
820 DATA 36, 30, 30, 30, 30, 3, 1.37
830 DATA 40, 36, 36, 36, 36, 3, 1.43
840 DATA 32, 31, 30, 30, 27, 3, 1.38
850 DATA 36, 32, 30, 30, 30, 30, 30
860 DATA 40, 36, 36, 36, 36, 36, 36
870 DATA 32, 32, 31, 30, 30, 28, 27
880 END
RUN
Enter the number of logs? 3
Enter the number of measured diameters on each log? 7
Enter section length (m.) ? .5

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		LOG VOLUMES (m3)					
No	of Log	VTRUE	VHUB	VSMAL	VHOS	VNEW	VCENT
	1	.2246257	.2120532	.2587049	.2650664	.2276037	.2246525
	2	.3112255	.3053565	.34117	.3816956	.3172943	.3161256
	3	.2137713	.2120532	.2065162	.2693664	.2102075	.2083658

		RESIDUALS OF THE LOG VOLUMES (m3)				
No	of Log	RHUB	RSMAL	RHOS	RNEW	RCENT
	1	-.0125726	0.034079	0.040441	-.224626	0.000027
	2	-.0058690	0.029944	0.070470	-.311226	0.004900
	3	-.0017181	-.007255	0.055595	-.213771	-.005405

		AVERAGE RESIDUALS OF THE METHODS				
		RHUB	RSMALL	RHOS	RNEW	RCENT
		-.0067199	0.018923	0.055502	-.249874	-.000160

		t-statistics of the methods				
		t-HUB	t-SMAL	t-HOS	t-NEW	t-CENT
		-2.12510	1.43973	6.40245	-8.10350	-0.05360

Ok

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